8th International Colloquium on Bluff Body Aerodynamics and Applications
Northeastern University, Boston, Massachusetts, USA
June 7 - 11, 2016

BBAA VIII

Comparison of Structural Mode Effects on Bending and Tension Dominated Flexible Cylinders in VIV

Ersegun D. Gedikli[a], Jason M. Dahl[b]

[a] *University of Rhode Island, Narragansett, RI, USA, egedikli@uri.edu*
[b] *University of Rhode Island, Narragansett, RI, USA, jmdahl@uri.edu*

SUMMARY:
Effects of structural mode shape excitation are investigated for the vortex-induced vibration of tension-dominated and bending-dominated cylinders. In the bending-dominated experiments, the test cylinder natural frequency is tuned such that the natural frequency ratio between in-line and cross-flow motion is fixed with a ratio of 2:1, while the mode shape ratio is allowed to vary with values of 1:1, 2:1, and 3:1. In the tension-dominated cylinder experiments, an initial tension was given to the cylinder, such that the first three modes may be excited in the in-line direction depending on flow speed. In both cases, the cylinder is not capable of oscillating with an even mode in the in-line direction with a 2:1 oscillation frequency ratio due to symmetric loading. Mode transition is observed in the tension dominated case and mode transitions in the bending-dominated cylinders demonstrate both 1:1 and 2:1 frequency excitations depending on the structural characteristics of the cylinder.

Keywords: Vortex-induced vibration; Flexible cylinder; Dynamic response; Modal decomposition

1. INTRODUCTION

Vortex-induced vibration (VIV) is a fluid-structure interaction prevalent in offshore structures where the interaction between vortex shedding and structural motions lead to amplitude-limited motions that can have a significant effect on structural fatigue and offshore operations. Understanding the fundamental behavior of this phenomenon is critical for designing structures to handle the effects of these vibrations, predicting these vibrations in offshore operations, and designing mitigation systems to prevent or limit these vibrations.

A number of reviews (Sarpkaya, 2004; Williamson and Govardhan, 2004) document much of the work devoted to the study of vortex-induced vibrations. Many of these fundamental studies focus on the vibration behavior of elastically mounted rigid bodies as canonical problems to understand the fundamental fluid-structure interaction problem. Studying the behavior of a flexible cylinder undergoing VIV is significantly more complex as a canonical problem due to multiple structural modes that may be excited in multiple directions, e.g. cross-flow (CF), in-line (IL), and torsional. Recent controlled laboratory experiments on flexible structures have investigated the dynamic response of low and high mode number cylinders (Gedikli and Dahl, 2014, Huera-Huarte and Bearman, 2009, Passano et al., 2010, Trim et al., 2005). Field experiments (Vandiver and Jong, 1987, Lie and Kaasen, 2006, Vandiver et al, 2005 and 2009) have investigated the multi-modal response of long cylinders in uncontrolled sheared currents.

Dahl et al. (2006) and Kang et al. (2013) investigated the role of separate mode excitations using elastically mounted, rigid cylinders allowed to move in two degrees of freedom. The effect of mode frequency ratio between IL and CF motion was modeled by tuning IL and CF natural frequencies on a rigid cylinder, hence the spatial mode shape of the cylinder did not affect the excitation of the body. Dahl et al. (2006) observed figure eight type motions of the body when natural frequency ratios between IL and CF directions were between 1:1 and 2:1, while Kang et al. (2013) observed teardrop shape motions when the natural frequency ratio is below 1:1. Dahl et al. (2007) also demonstrated that there is a natural preference for figure eight motions to move upstream (denoted counter-clockwise) at the top and bottom of the orbit, which may naturally occur for an elastically-mounted, rigid cylinder, however may have more difficulty occurring in a flexible structure undergoing multi-mode excitation, where this preferred condition may not be allowed at every point along the length of the structure.

The present study aims to clarify the behavior of a flexible cylinder in a systematically designed experiment to see the effect of structural mode shape on the excitation. In this study, two sets of experiments (bending-dominated and tension-dominated) are performed. In the bending-dominated experiments, the natural frequencies of the flexible structure are tuned to be 2:1 (IL to CF) while the modes associated with these frequencies have a ratio of 1:1, 2:1, 3:1 (IL to CF). The choice of fixing the natural frequencies to be 2:1 attempts to match the natural frequency of the structure with the natural dominant frequencies of vortex shedding in the wake of the structure. The structural mode shapes assumed in the design of the experiment are based on the analytic solution to the equation of motion for an un-tensioned beam with simply supported ends, assuming the beam vibrates in vacuum. In the tension-dominated cylinder experiments, a flexible cylinder with similar aspect ratio is tensioned with an initial tension and structural mode shapes are assumed based on the analytic solution of a tensioned string with simply supported ends. Tests are performed for a variety of flow speeds with very small increments allowing excitation of the structure due to the fluid-structure interaction. The present study demonstrates that 2:1 mode shape cannot regularly be excited due to the symmetric distribution of the drag force along the length of the structure.

2. METHODOLOGY

Experiments were conducted in a re-circulating open water flow channel with uniform flow speed in the test section. The flow speed was varied from 0.1 to 0.7 m/s. The test section of the flow channel has glass-viewing walls on all sides and the bottom, allowing for visual access to the test section. The dimensions of the test section were 38 by 48 cm.

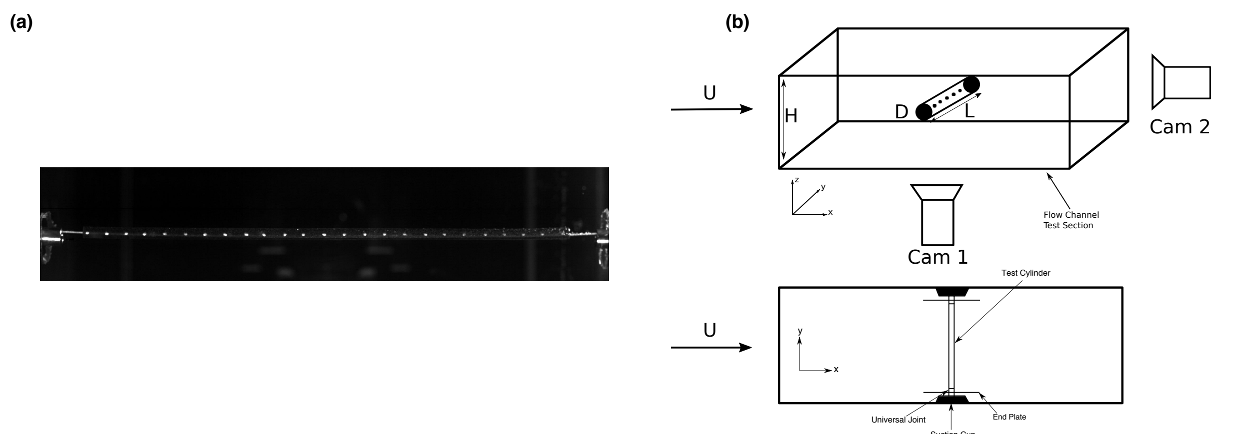

Figure 1. (a) bending-dominated test cylinder, raw image (b) general experimental setup, isometric view and top view.

Two types of cylinders were used as models to produce a bending-dominated cylinder and a tension-dominated cylinder (e.g. see Fig. 1(a)). For the bending-dominated model cylinder, a plastic beam was molded inside of a flexible urethane material to achieve the desired structural characteristics. For the tension-dominated cylinder, a uniform flexible rubber material was stretched across the test section and then held in the stretched orientation to define the baseline frequency characteristics of the cylinder. Cylinders were mounted horizontally in the water channel specifically to limit surface effects and asymmetry to the loading. Horizontal mounting was achieved by using strong suction cups connected to the ends of the cylinders to affix the cylinders to the glass walls of the test section. The test cylinders were marked with 23 to 25 white dots depending on each test cylinder, where the dots were evenly spaced along the length. Two synchronized high-speed cameras captured time-resolved perpendicular images of the cylinder and motion tracking was used to determine the IL and CF oscillation of the cylinder. The high-speed cameras recorded the motion of the cylinder with a frame rate of 250 Hz allowing for capture of the relevant mode frequencies of interest. As seen in Table 2, the frame rate was not sufficient to capture all of the first three mode frequencies for all cylinders, however it was sufficient to capture the first three mode frequencies for cylinder 3, where the first three modes were of interest. A schematic drawing of the experimental setup is shown in Figure 1(b).

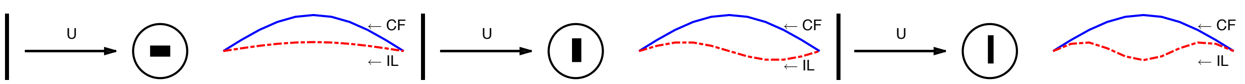

Figure 2. Ideal excitation of the bending-dominated structural modes. The left image shows the 1:1 modal response (cylinder 1), center image shows the 2:1 modal response (cylinder 2), right image shows the 3:1 modal response (cylinder 3). Blue continuous line shows ideal CF cylinder response; red dashed line shows ideal IL cylinder response.

Table 1. Non-dimensional Parameters

Non-dimensional Amplitude	$A^* = \frac{A}{D}$	**Mass Ratio**	$m^* = \frac{4m}{\rho\pi L D^2}$
Reynolds Number	$Re = \frac{UD}{\nu}$	**Aspect Ratio**	$AR = \frac{L}{D}$
Nominal Reduced Velocity	$V_{rn} = \frac{U}{f_n\, D}$	**Blockage Ratio**	$B = \frac{D}{H}$
Reduced Velocity	$V_r = \frac{U}{f\, D}$		

Relevant normalized parameters used in this study are given in Table 1, where U is the flow velocity, D is the cylinder diameter, f_n is the vacuum fundamental natural frequency in CF direction, f is the observed oscillation frequency in the CF direction, ν is the kinematic viscosity of fresh water, H is the height of the cylinder from the bottom of the tank, m is the cylinder mass, and ρ is the water density. The physical dimensions, non-dimensional values, and cylinder characteristics are given in Table 2. Bending dominated test cylinders are specified as cylinder 1, cylinder 2 and cylinder 3 with 1:1, 2:1, and 3:1 mode ratios, respectively, whereas the tension dominated test cylinder is specified as cylinder 4.

Since the natural frequency of each bending-dominated cylinder is tuned through structural characteristics, to achieve different mode shapes, beams of different dimensions are molded inside the urethane cylinder. Table 2 gives the structural characteristics of each beam, while

Figure 2 shows a schematic drawing of each beam cross-section and the associated ideal structural mode shapes that one would expect to excite from 2:1 (IL:CF) forced excitation.

Table 2. Cylinder Characteristics

		Cylinder 1	Cylinder 2	Cylinder 3	Cylinder 4
Cylinder Type		Bending	Bending	Bending	Tension
Cylinder Material		Urethane	Urethane	Urethane	Rubber
Beam Material		Plastic	Plastic	Plastic	None
Diameter (mm)		6.35	6.35	6.35	6.35
In-line beam width (mm)		1.27	2	2.25	None
Cross-flow beam width (mm)		2.5	0.04	0.508	None
Reynolds Number Range		1500-5500	1700-5400	1600-4700	650-3500
Blockage Ratio		1.66	1.66	1.66	1.66
Aspect Ratio		41	41	41	41
Mass Ratio		1.1	1.05	1.02	3.7
Tension (N)		None	None	None	0.15
In-line natural frequency (Hz)	**Mode 1**	34	7	1.82	3
	Mode 2	136	28	7.3	6
	Mode 3	306	63	16.4	12
Cross-flow natural frequency (Hz)	**Mode 1**	17	14	8.2	3
	Mode 2	68	56	32.8	6
	Mode 3	153	126	73.8	12

3. RESULTS

In bending and tensioned cylinder experiments, a 2:1 frequency response, characterized by figure eight or crescent shaped cylinder motions, was observed for the majority of flow conditions, however under some flow conditions, a 1:1 frequency response was also observed, characterized by teardrop shaped motions with small IL response. Modal decomposition of the response of the tested cylinders was used to analyze the modal contributions to the cylinder response. The modal contribution of the first three proper orthogonal modes (POMs) were calculated for cylinders 2, 3, and 4 since the first 3 POMs correspond to approximately 90% of the total oscillation energy in all the cases tested. However, since cylinder 1 never reaches the second mode frequency in the IL and CF directions at the speeds tested, the modal contributions for this case are not shown.

3.1. Bending Dominated Cylinders

3.1.1. Maximum amplitude response

Figure 3 shows a comparison of the normalized maximum amplitude responses of each bending and tension dominated cylinder as a function of reduced velocity. The 1:1 and 2:1 responses are typical for flexible structures, displaying an increase in amplitude as reduced velocity increases (Huera-Huarte and Bearman, 2009). Due to the squared relation of mode number to natural frequency in a simply-supported beam, the flow speeds do not reach a high enough value to excite any higher mode frequencies in the CF direction, although multiple modes are excited in cylinders 2 and 3. Cylinder 2 shows an overall larger maximum CF response than cylinders 1 and 3, however cylinder 1 shows a significantly larger IL response than cylinders 2 and 3. Cylinder 3 shows two distinct branch responses for reduced velocities between 5.2 and 5.6. The lower amplitude branch (labeled d) corresponds to a dominant frequency excitation in the IL direction that is the same frequency as the CF direction, resulting in a tear-drop shaped Lissajous figure. The larger amplitude response (labeled c) corresponds to excitation in the IL direction with twice the frequency of the CF direction, resulting in a figure eight type response. The tension dominated cylinder (cylinder 4) displays a similar two branch response, however the reason for the branched response differs from cylinder 3. The upper response (labeled a) follows a similar increase in amplitude as reduced velocity increase, similar to cylinders 1 and 2, however at high flow speeds, the cylinder transitions from a dominant first mode excitation to a dominant second mode excitation in the CF direction, as the shedding frequency approaches the second mode frequency of the cylinder. This results in the response dropping to the lower amplitude response (labeled b) as seen in Figure 3.

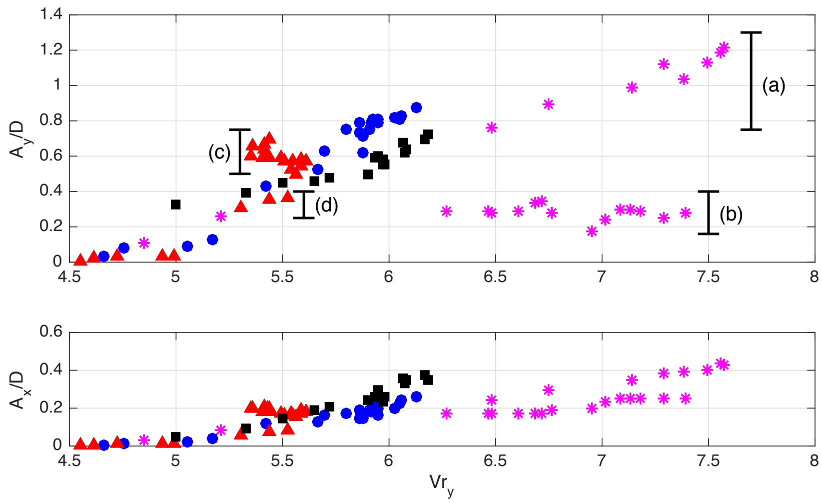

Figure 3 Normalized amplitude response as a function of CF reduced velocity. Top image shows the cross-flow amplitude response and bottom image shows the in-line amplitude response. Black square - cylinder 1, blue circle - cylinder 2, red triangle - cylinder 3, magenta star - cylinder 4.

3.1.2. Spanwise response: Cylinder 1

An example response of the 1:1 cylinder is shown in Figure 4, characterized by figure eight motion with a 2:1 ratio between the IL and CF response frequencies. This example shows the spanwise shape of the response, frequencies of the IL and CF responses, and Lissajous response measured at the center point of the cylinder for $Vr_n = 6.8$. This response is typical for all reduced velocities. The response compares well with what is typically seen for an elastically mounted rigid cylinder undergoing two degree of freedom VIV (Dahl et al., 2006). The observed frequency of the response demonstrates the typical effective natural frequency resonance due to changes in the forces in phase with acceleration of the cylinder. The observed spanwise response shows a first mode, symmetric spatial shape as expected.

3.1.3. Spanwise Response: Cylinder 2

An example response for cylinder 2 at $Vr_n = 8.26$ is shown in Figure 5. Cylinder 2 demonstrates a frequency response where the IL motion has a frequency twice the CF motion frequency, similar to cylinder 1, however the IL spanwise shape does not match the structural second mode shape for a simply supported beam. As shown in Figure 5, the IL shape of the vibration is primarily first mode. There is a slight asymmetry to the shape associated with a small second mode component, however the dominant shape is the first mode, despite the frequency of the response being well above the second structural natural frequency.

Figure 6 shows the modal decomposition of the first three POMs associated with cylinder 2's spatio-temporal response, showing the percentage of modal contribution as a function of nominal reduced velocity. The figure also shows the normalized shape functions for the computed POMs at Vr_n = 8.26, to show an example of the resulting mode shapes. Since the proper orthogonal decomposition (POD) ranks the modes based on the most energetic modes, the shape functions are labeled as POM 1, POM 2, and POM 3, however the order of these modes do not necessarily correspond with the first three Fourier modes of the beam. The bottom figures of Figure 6 sorts the modes based on visual inspection of the POM shape functions, such that the reported 1st mode corresponds with a symmetric half sine mode shape, the 2nd mode corresponds with an asymmetric full sine mode shape, and the 3rd mode corresponds with a symmetric one and a half sine shape, to demonstrate how the dominance of different mode shapes change with reduced velocity. For example, depending on the reduced velocity, POM 2 may correspond with an asymmetric 2nd mode shape or a symmetric 3rd mode shape, depending on which mode is more energetic at a particular speed. The first three POM shape functions are similar for all cylinders, so are only shown for cylinder 2 in Figure 6. The 1st mode shape is shown to be dominant for all the tested flow speeds in CF and IL directions. The CF direction shows nearly zero contribution from higher modes, while the IL direction shows contributions from the second and third modes that account for approximately 20-30% of the energy in the response. The asymmetric second mode IL is significant at the middle reduced velocities, while the symmetric third mode becomes dominant at higher reduced velocities despite the response frequency occurring closest to the second structural mode.

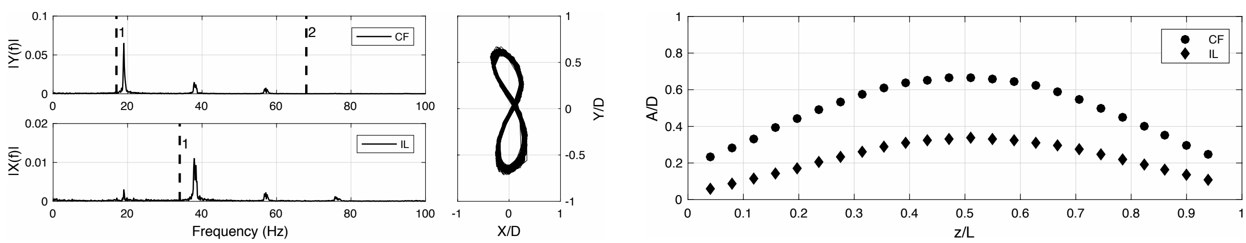

Figure 4. Example response of cylinder 1 (1:1 expected mode shape) at Vr_n = 6.8. Left image shows the frequency response of the center point on the cylinder with corresponding Lissajous shape. Vertical dashed lines on the left show the 1st and 2nd mode natural frequency in vacuum. Right image shows corresponding amplitude response as a function of the cylinder span.

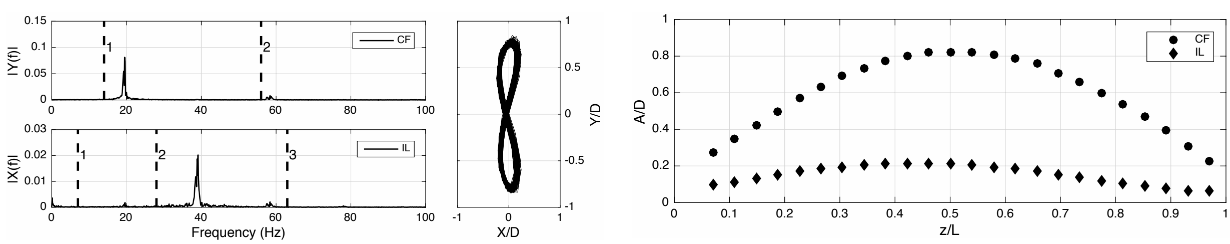

Figure 5. Example response of cylinder 2 (2:1 expected mode shape) at Vr_n = 8.26. Left image shows the frequency response of the center point on the cylinder with corresponding Lissajous shape. Vertical dashed lines on the left illustrate the 1st, 2nd, and 3rd natural frequency in vacuum. Right image shows corresponding amplitude response as a function of the cylinder span.

3.1.4. Spanwise Response: Cylinder 3

In contrast to cylinders 1 and 2, cylinder 3 displays a distinctly different behavior over the range of flow speeds tested. The cylinder displays two different response branches: 1) the IL frequency is shown to be twice the CF frequency, as with the other cylinders, 2) the IL frequency is observed to be equal to the CF frequency. Figure 7 shows two examples of these response regions for nominal reduced velocities of 7.3 and 8.4. For the reduced velocity of 7.3 (top figures), the cylinder motion resembles a squished tear drop shape with a very small IL motion and a large CF motion. This is consistent with the observations of Kang et al. (2013) when the natural frequency ratio between IL and CF directions of a two degree of freedom rigid cylinder is less than 1. In this case, the cylinder oscillates with a dominant 1:1 frequency ratio as seen in the spectrum. The response changes as the nominal reduced velocity increases, leading to a transition to figure eight shape motion and a dominant 2:1 frequency ratio between IL and CF motion. The bottom of Fig. 7 shows this transition for a reduced velocity of 8.4. In both cases, the maximum spanwise response is nearly symmetric.

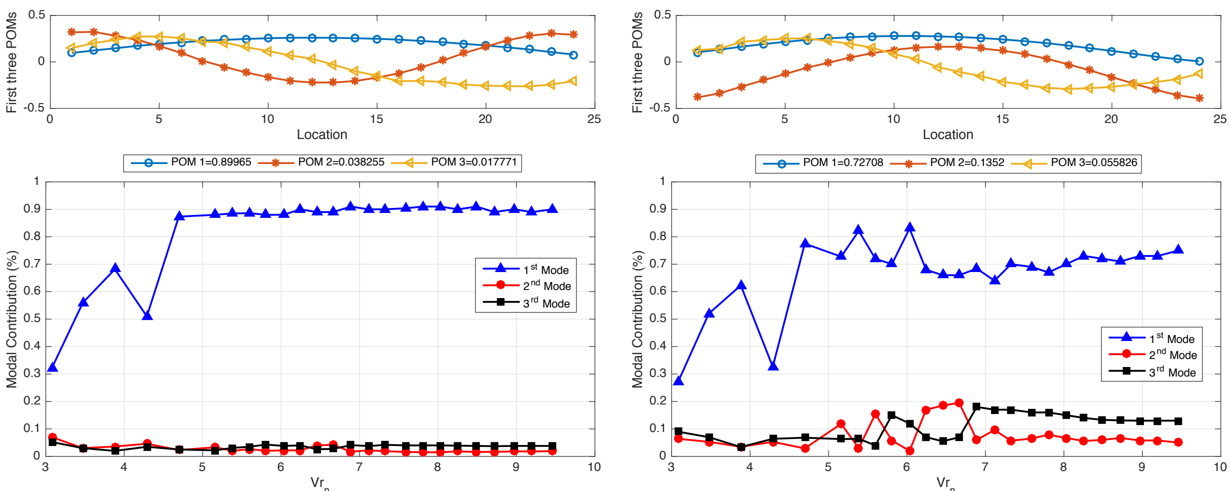

Figure 6. First three modes from the POD of cylinder 2 response. Top left – Normalized ranked POM shape functions at Vr_n=8.26 in CF. Top right – same as top left for IL direction. Bottom left – Structural modes excited in CF based on sorting of POMs as a function of reduced velocity, Bottom right – same as bottom left for IL direction.

Figure 8 shows the modal decomposition for cylinder 3. Again, a first structural mode shape is dominant for all speeds in both directions, despite the beam being tuned to excite the third structural mode IL. The third mode is observed to be predominantly larger than the 2nd mode for nearly all flow speeds, even when the response is characterized by a 1:1 frequency excitation.

3.2. Tension Dominated Cylinder: Cylinder 4

The tension-dominated experiments were conducted over a similar Reynolds number range with a similar aspect ratio cylinder, and it was found that as the flow speed was increased with small increments, the cylinder first responded with 1:1 mode shape, then transitioned to oscillating with 3:2 mode shape as the shedding frequency approached higher natural frequencies, effectively skipping the second mode in the IL direction.

Figure 9 shows two example responses corresponding to nearby flow speeds where the mode transition occurs. At the lower flow speed of Vr_n=12.6 (top row), the cylinder oscillates with dominant first mode in both IL and CF directions with a figure eight motion, while at a higher flow speed with Vr_n=14.3 (bottom row), the cylinder oscillates with significant first and third mode contributions in the IL direction and a dominant second mode in the CF direction resulting in an irregular Lissajous orbit, shown for the quarter span point since the center point for this response occurs at a node for the CF response.

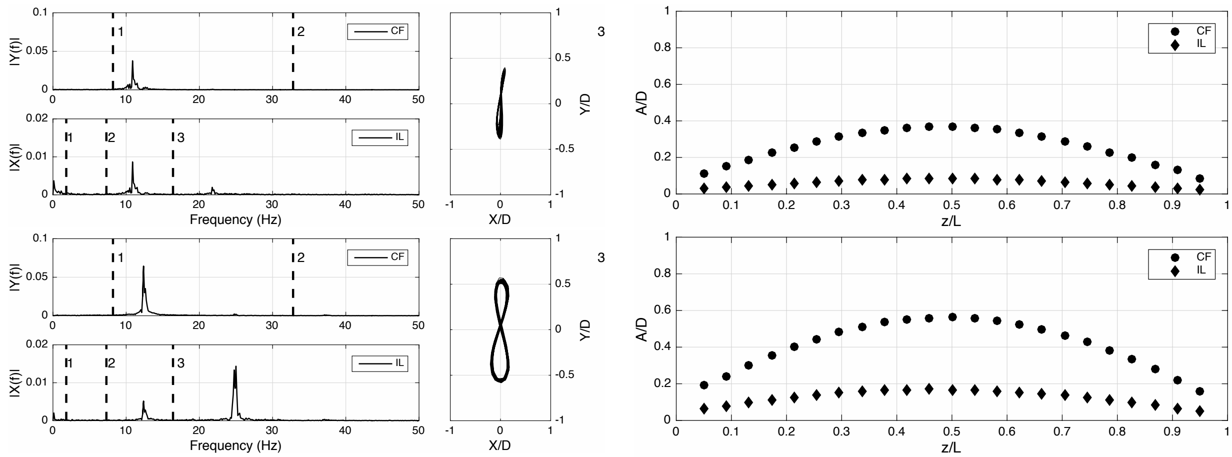

Figure 7 Example responses of cylinder 3 (3:1 expected mode shape) for Vr_n = 7.3 (top row) and Vr_n = 8.4 (bottom row). Left: Frequency response of the center point on the cylinder with corresponding Lissajous shape. Vertical dashed lines show the 1st, 2nd, and 3rd mode natural frequency in vacuum. Right: Maximum amplitude response as a function of cylinder span.

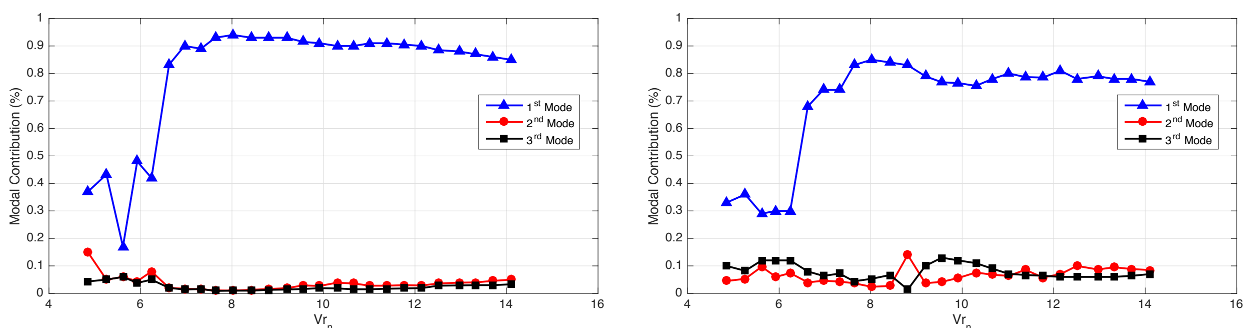

Figure 8. First three modes from the POD of cylinder 3 response. Left – Structural modes excited in CF based on sorting of POMs as a function of reduced velocity, Right – same as left for IL direction.

Figure 10 shows examples of the modal decomposition for the tension-dominated cylinder. The transition between first mode and second mode is distinct, but in the region where the first modes are dominant, the response contributions are similar to the 1:1 bending dominated cylinder (cylinder 1). Once the response transitions to a higher mode response, the second and third modes contribute more significantly to the response, however the second mode in the IL direction is again observed to never be dominant.

4. DISCUSSION/CONCLUSION

In comparison, the tension-dominated and bending-dominated systems in this study demonstrate one common behavior: despite frequency excitation in CF that is twice the frequency in the IL direction and tuning natural frequencies to have specific mode shapes, it is difficult or not possible to significantly excite an asymmetric second mode shape in the IL direction. Vandiver and Jong (1987) observed a similar behavior in field experiments, attributing this behavior to the symmetric distribution of the drag force over the cylinder in a uniform flow. If one considers a uniform distribution of force over the span of a simply-supported beam, where the amplitude of the force is a harmonic function applied with a frequency equal to the second natural frequency, one finds that the structure will respond with a frequency equal to forcing frequency, but the spanwise response will have a symmetric shape similar to the first mode shape. In fact, for any frequency associated with an even mode, the spanwise shape will be similar to the next lowest odd mode shape. Even modes could certainly be excited in the case of sheared flow, where an asymmetry of the flow speed would result in an asymmetry to the distributed drag load. This is also significant to flexible cylinder studies conducted vertically in a towing tank or water tunnel. In conditions where a flexible cylinder pierces the water surface, a slight asymmetry may occur in the loading of the structure, which would demonstrate asymmetric mode excitation that would not typically occur if the loading was purely symmetric (for example if the cylinder is oriented horizontally in the water column). This has general relevance in understanding responses observed in field or lab experiments studying the response of flexible cylinders.

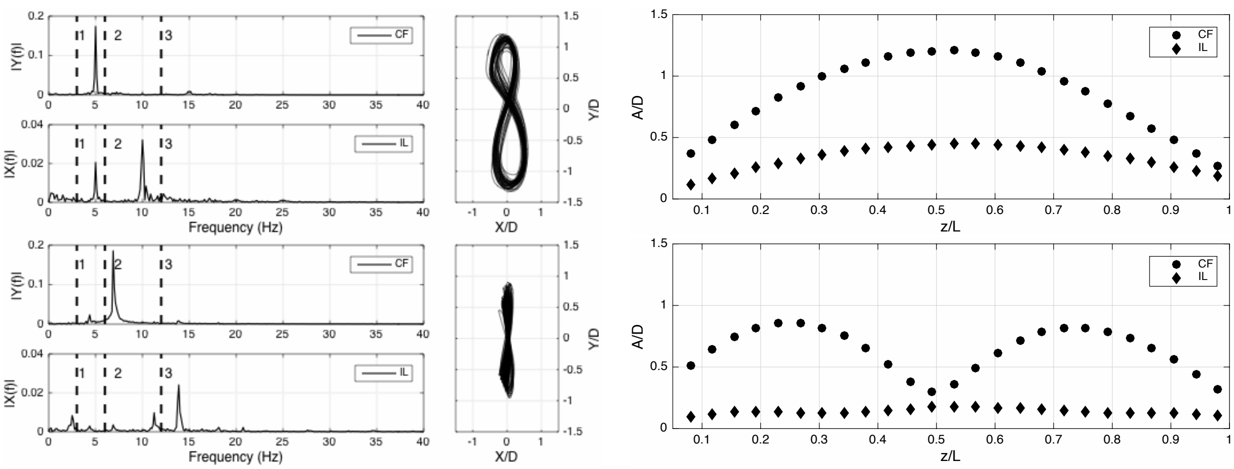

Figure 9. Example response for cylinder 4, tension dominated, at Vr_n = 12.6 (top) and Vr_n = 14.3 (bottom). Top Left: Frequency response of the center point on the cylinder with corresponding Lissajous shape. Vertical dashed lines show the 1st, 2nd, and 3rd mode natural frequency in vacuum. Top/Bottom Right: Maximum spanwise amplitudes. Bottom Left: Frequency response at z/L=0.25 on the cylinder at Vr_n=14.3 with Lissajous shape.

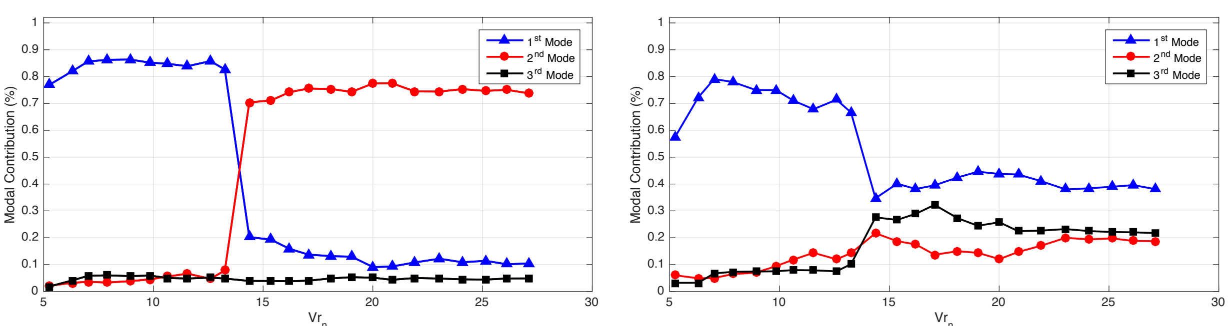

Figure 10. First three modes from the POD of cylinder 4 response. Left – Structural modes excited in CF based on sorting of POMs as a function of reduced velocity, Right – same as left for IL direction.

In the particular experiments shown, additional interesting behaviors are observed. For example, cylinder 3 is tuned such that the first mode IL will correspond with the forcing frequency from vortex shedding in the transverse direction, while the third structural mode will correspond with the vortex shedding frequency in the IL direction. Despite this tuning, the IL direction undergoes a resonant response with dominant first mode shape (due to the loading distribution as described above). This is interesting, however, since in order to oscillate with the observed frequency and mode, a linear treatment of the frequency response and adjustment of the effective natural frequency would require an extremely large negative added mass, since the frequency of oscillation in the IL direction is so far from the natural frequency associated with the first mode. This is likely not possible and the frequency transitions observed for cylinder 3 are more likely due to non-linear resonant conditions from the coupling of vortex shedding effects on the CF and IL response. It is necessary to investigate forces and wake structures on these flexible structures in order to understand this relationship.

Finally, the transition between a 1:1 mode shape response and 2:3 mode shape response seen in the tensioned cylinder is not necessarily unique to the tensioned cylinder, since the natural frequency relation for the bending-dominated cylinder requires that natural frequencies are further spaced from one another. Due to limitations of the flow channel, higher speeds could not be tested to see if the transition to higher modes follows a similar behavior for the bending-dominated systems.

ACKNOWLEDGEMENTS

Ersegun Deniz Gedikli, recipient of a travel fellowship to attend BBAA VIII, gratefully acknowledges the support of the BBAA VIII organization and its sponsors.

REFERENCES

Dahl, J.M., Hover, F.S., and Triantafyllou, M.S., 2006. Two-degree-of-freedom vortex-induced vibrations using force assisted apparatus. Journal of Fluids and Structures, 22:807-818.

Dahl, J.M., Hover, F.S., Triantafyllou, M.S., Dong, S., and Karniadakis, G.E., 2007. Resonant vibrations of bluff bodies cause multi-vortex shedding. Physical Review Letters, 99 (144503).

Gedikli, E.D., Dahl, J.M., 2014. Mode Shape Variation for a Low-Mode Number Flexible Cylinder Subject to Vortex-Induced Vibrations, ASME 2014 33rd International Conference on Ocean, Offshore and Arctic Engineering. American Society of Mechanical Engineers, pp. V002T008A071-V002T008A071.

Huera-Huarte, F.J., Bearman, P.W., 2009a. Wake structures and vortex-induced vibrations of a long flexible cylinder – part 1: dynamic response. Journal of Fluids and Structures 25, 969–990.

Kang, Z., Jia, L., 2013. An experiment study of a cylinder`s two degree of freedom VIV trajectories. Ocean Engineering, 70:129-140.

Lie, H., Kaasen, K.E., 2006. Modal analysis of measurements from a large scale VIV model test of a riser in linearly sheared flow. Journal of Fluids and Structures, 22, 557–575.

Passano, E., Larsen, C.M., and Wu J., 2010. VIV of free spanning pipelines: Comparison of response from semi-empirical code to model tests. Proceedings of the ASME 2010 29th International Conference on Ocean, Offshore and Arctic Engineering. Shanghai, China

Sarpkaya, T., 2004. A critical review of the intrinsic nature of vortex-induced vibrations. Journal of Fluids and Structures, 19:389-447.

Trim, A.D., Braaten, H., Lie, H. and Tognarelli, M.A., 2005. Experimental investigation of vortex-induced vibration of long marine risers. Journal of Fluids and Structures, 21:335-361.

Vandiver, J.K., and Jong, J.-Y., 1987. The relationship between in-line and cross-flow vortex-induced vibration of cylinders. Journal of Fluids and Structures, 1:381-399.

Vandiver, J.K., Marcollo, H., Swithenbank S., and Jhingran V., 2005. High mode number vortex-induced vibration field experiments. Offshore Technology Conference, Paper Number 17383, Houston, Texas.

Vandiver, J.K., Jaiswal, V., Jhingran,V., 2009. Insights on vortex- induced, traveling waves on long risers. Journal of Fluids and Structures 25,641–653

Williamson, C.H.K., and Govardhan, R., 2004. Vortex-induced vibrations. Annual Review of Fluid Mechanics, 36:413-455.